Components of random geometric graphs

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The random geometric graph (RGG) is obtained by placing n vertices uniformly at random in a bounded region of R^d and connecting any two vertices distant at most r apart. We discuss large-n asymptotics with $r = r_n$ a specified sequence.

Given a positive integer k, let $S_{n,k}$ be the number of components of order k in this graph, and let $S_n := \sum_k S_{n,k}$, the total number of components. Let $L_n := \max\{k : S_{n,k} > 0\}$, the order of the largest component.

In the 'thermodynamic limit' where $nr_n^d \to c \in (0, \infty)$, a law of large numbers and central limit theorem were already known for $S_{n,k}$, and for S_n, and for L_n. We discuss newer results of this type in the 'dense limit' where $nr_n^d \to \infty$ slowly.

In a related result, we determine the large-λ asymptotics for the probability that the origin lies in a cluster of order k in a Poisson Boolean model with intensity λ.

[1] M. D. Penrose and X. Yang (2023) On k-clusters of high-intensity random geometric graphs. arXiv:2209.14758